

Assignment of Chapter 1

1. Given the joint distribution of X and Y , please determine:

	$X = 0$	$X = 1$
$Y = 0$	$\frac{1}{6}$	$\frac{1}{3}$
$Y = 1$	$\frac{1}{6}$	$\frac{1}{3}$

- (a) $H(X), H(Y)$;
- (b) $H(X|Y), H(Y|X)$;
- (c) $H(X, Y)$;
- (d) $I(X; Y)$.

2. A discrete random variable X takes values in $\{0,1,2,3\}$ with

$$P(X = 0) = P(X = 1) = P(X = 2) = P(X = 3) = \frac{1}{4}.$$

The random variable X is transmitted through a communication system, producing an observation Y at the receiver. The receiver can use Y to estimate $\hat{X}$ of X . The error probability is known as

$$P_e = P(X \neq \hat{X}) = \frac{1}{8}.$$

- (a) Please use Fano's inequality, give an upper bound on $H(X|Y)$.
- (b) Please give a lower bound on $I(X; Y)$ based on

$$I(X; Y) = H(X) - H(X|Y).$$

- (c) Please explain how the upper bound on $H(X | Y)$ changes as P_e increases, and what this implies for the communication system.

3. Let the true probability distribution of a random variable X be

$$P(X) = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8} \right\},$$

but due to an inaccurate estimation of the source probability distribution, the receiver assumes that the probability distribution is

$$Q(X) = \left\{ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\}.$$

(a) Please calculate the relative entropy

$$D(P(x), Q(x)) = \sum_x P(x) \log_2 \frac{P(x)}{Q(x)}.$$

(b) please prove

$$D(P(x), Q(x)) = H(Q(x)) - H(P(x)).$$

(c) Please prove the chain rule for relative entropy

$$D(P(x, y), Q(x, y)) = D(P(x), Q(x)) + D(P(y|x), Q(y|x)).$$